Cambridge Public Schools
Precalculus Honors/College Prep Curriculum Map
2013 – 2014

The 2013-2014 school year will be the start of the district's full implementation to the new 2011 Massachusetts Mathematics Curriculum Framework for Grades 9 through 12.

Precalculus combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college.

In Precalculus, instructional time should focus on four critical areas:

(1) extend work with complex numbers;
(2) expand understanding of logarithms and exponential functions;
(3) use characteristics of polynomial and rational functions to sketch graphs of those functions; and
(4) perform operations with vectors.

(1) Students continue their work with complex numbers. They perform arithmetic operations with complex numbers and represent them and the operations on the complex plane. Students investigate and identify the characteristics of the graphs of polar equations, using graphing tools. This includes classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

(2) Students expand their understanding of functions to include logarithmic and trigonometric functions. They investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This includes the role of e, natural and common logarithms, laws of exponents and logarithms, and the solutions of logarithmic and exponential equations. Students model periodic phenomena with trigonometric functions and prove trigonometric identities. Other trigonometric topics include reviewing unit circle trigonometry, proving trigonometric identities, solving trigonometric equations, and graphing trigonometric functions.

(3) Students investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. They determine zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Students translate between the geometric description and equation of conic sections. They deepen their understanding of the Fundamental Theorem of Algebra.

(4) Students perform operations with vectors in the coordinate plane and solve practical problems using vectors. This includes the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

Standards for Mathematical Practice

The 2011 framework introduces Standards for Mathematical Practice. These standards complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. These standards are the same at all grades from Pre-Kindergarten to 12th grade. These eight practices can be clustered into the following categories as shown in the chart below:

Habits of Mind of a Productive Mathematical Thinker: MP.1: Make sense of problems and persevere in solving them. MP.6: Attend to precision.	***Reasoning and Explaining*** MP.2: Reason abstractly and quantitatively. MP.3: Construct viable arguments and critique the reasoning of others
	Modeling and Using Tools MP.4: Model with mathematics. MP.5: Use appropriate tools strategically.
	Seeing Structure and Generalizing MP.7: Look for and make use of structure. MP.8: Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice in High School

The Pre-K – 12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The following lists examples of what the practice standards look like in High School.

Standards	***Explanations and Examples***
Students are expected to: **1. Make sense of problems and persevere in solving them**.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Standards	***Explanations and Examples***
Students are expected to: **2. Reason abstractly and quantitatively.**	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
Students are expected to: **3. Construct viable arguments and critique the reasoning of others.**	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Students are expected to: **4. Model with mathematics.**	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Standards	*Explanations and Examples*
Students are expected to: **5. Use appropriate tools strategically.**	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Students are expected to: **6. Attend to precision.**	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Students are expected to: **7. Look for and make use of structure.**	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

Standards	***Explanations and Examples***
Students are expected to: **8. Look for and express regularity in repeated reasoning**.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

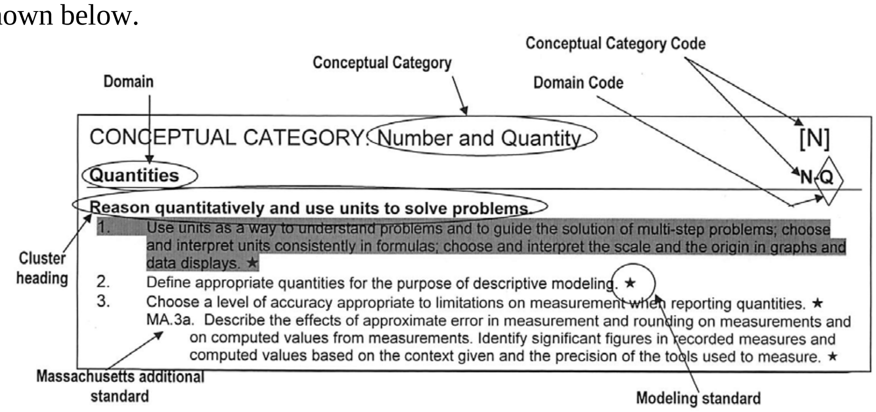
Organization of the High School Content Standards in the 2011 framework
The high school content standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses, such as calculus, advanced statistics, or discrete mathematics, is indicated by a (+) symbol. Because the standards for this course are (+) standards, students selecting this Precalculus course should have met the college and career ready standards.

The high school standards are listed in **conceptual categories**:

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Modeling (★)
- Geometry (G)
- Statistics and Probability (S)

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Similar to the grade level content standards, each conceptual category (except Modeling, see explanation following the illustration) is further subdivided into several domains, and each domain is subdivided into clusters.

Standards Identifiers/Coding
High school content standards are identified first by conceptual category, rather than by grade as for pre-kindergarten through grade 8 content standards. The code for each high school standard begins with the identifier for the conceptual category code (N, A, F, G, S), followed by the domain code, and the standard number, as shown below.



Unique Massachusetts Standards
Standards unique to Massachusetts are included in the appropriate domain and cluster and are initially coded by "MA." For example, the Massachusetts standard **MA.N.Q.3a** is identified with "MA" indicating a *Massachusetts* addition, "N" indicating it is from the *Number and Quantity* Conceptual Category, "Q" indicating the *Quantity* domain, and "3a" indicating that it is a further specification to the third standard in that domain.

The star symbol (★) following the standards in the illustration indicates those are also Modeling standards. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Time Period 1: Functions

11 days + 2 days for review and testing.

Notes:

- Prior Knowledge: This unit largely serves as a review of parent functions and graph transformations. Students should be familiar with most parent functions (linear, quadratic, cubic, square root, absolute value, exponential, logarithmic, inverse, sine, and cosine) from Algebra I and Algebra II. Students should also be able to apply their knowledge of graph transformations (horizontal and vertical dilations, reflections, and translations) in the form $y = \pm af(\pm b(x-h)) + k$ to all parent functions.
- Precalculus Focus: Students will be introduced to the greatest integer and logistic function. Instructional time should be used to help students recall and develop a variety of strategies to solve absolute value and polynomial equations. This time should also be used to help students develop skills to model and solve real-world situations with polynomial functions.
- Next Steps:
    - AP Calculus: Students should have a strong level of fluency with parent functions, graph transformations, and solving a variety of equations both with and without technology.
    - SAT Subject Test Math Level II: Coordinate geometry comprises 10-14% of this test. Topics in this unit that can be categorized as coordinate geometry include: plotting points, slope of parallel/perpendicular lines, equations of lines, determining intercepts, knowledge of graph transformations as applied to all parent functions, and graphing inequalities. Functions and Algebra comprises approximately 48-52% of this test. Topics in this unit that can be categorized as functions and algebra include: composition of functions, determine the maxima/minima, and piecewise functions.

Essential Concepts	Essential Questions
Modeling with Functions and Equation Solving	▪ How do we use functions to model real-life applications? ▪ How do we choose which function best fits the scenario?
Parent Functions, Function Analysis, and Graph Transformation	▪ How can we use characteristics of functions to analyze the 12 parent functions and other functions? ▪ How can we fluently translate between the algebraic and graphical representation of a function?
Function Compositions and Inverse Functions	▪ How do we perform operations and compositions with functions? ▪ How do we find the inverse of a function numerically, graphically, and algebraically?

Standards
- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities. ★

F-BF.1c (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* ★

Pacing:

# of Days	Topic	Scope of Standard	Textbook Section(s)
2-3	Modeling with Functions and Equation Solving	▪ Review model and solve a variety real-world situations with linear and quadratic functions. ▪ Ensure that students can decide which model best fits the scenario and identify the most efficient way to solve the problem.	1.1, 1.6
6	Parent Functions, Function Analysis, and Graph Transformation (Review)	▪ Review other parent functions, but focus on greatest integer and logistic function. ▪ Review horizontal and vertical dilations, reflections, and translations) in the form $y = \pm af(\pm b(x-h)) + k$ to all parent functions.	1.2, 1.3, 1.5
2	Function Compositions and Inverse Functions	▪ Use compositions to determine whether two functions are inverse functions or not. ▪ Find inverse functions numerically, graphically, and algebraically (focus on algebraic fluency).	1.4
2	Review and Assessment		Chapter 1

Time Period 2: Polynomial and Rational Functions

13 days + 2 days for review and testing.

Notes:

- Prior Knowledge: From Algebra II, students should be able to identify zeros when suitable factorizations are available, and use the zeros to construct a rough graph of the defined function. Students should also be able to rewrite rational expressions in different forms.
- Precalculus Focus: Students will construct polynomial graphs with zeros and end behavior, and apply limit notation to the end behavior of functions. The greater focus in Precalculus lies in understanding and applying the Fundamental Theorem of Algebra, Rational Root/Zero Theorem, Remainder Theorem, Upper/Lower Bound Tests and using them to solve higher-degree equations. Students will focus on dividing rational functions and graphing rational expressions using intercepts and behavior around horizontal and vertical asymptotes. Students will also use limit notation to describe behavior around vertical asymptotes. Students may or may not have had exposure to the Binomial Theorem in Algebra II, but should be exposed to it in Precalculus.
- Next Steps:
  - AP Calculus: Students should have a strong level of fluency for finding zeros of polynomials and limit notation.
  - SAT Subject Test Math Level II: Functions and Algebra comprises approximately 48-52% of this test. Topics in this unit that can be categorized as algebra include: add/subtract/multiply/ divide polynomials, factor polynomials (trinomials, difference of two squares, sum/difference of cubes), add/subtract/multiply/divide/simplify rational expressions, solve equations with rational expressions, Binomial Theorem. Topics in this unit that can be categorized as functions and algebra include: higher-order polynomial functions and rational functions. Numbers and Operations comprise 10-14% of this test. Topics in this unit that can be categorized as number and operations include: meaning of a limit, limit notation, determining limits graphically and algebraically, use a complex conjugate to write a complex number in standard form, perform mathematical operations on complex numbers

Essential Concepts	**Essential Questions**
Modeling with Linear, Quadratic, and Power Functions	▪ How do we know when to model a scenario with linear, quadratic, or power functions? ▪ How do we solve the corresponding equation?
Graphing Higher Degree Polynomials	▪ How do we sketch a graph of higher degree polynomials to include accurate end behavior and zeros?
Finding pure real and complex zeros of polynomial functions	▪ How do we find the complex (purely real and real/imaginary) zeros of a polynomial functions?
Operations with complex numbers (review), complex plane, modulus of complex numbers, distance and midpoint	▪ How do we use operations with complex numbers to find the modulus of complex numbers? ▪ How do we find the distance and midpoint between two points on a complex plane?
Graphing and analyzing graphs of rational functions, limits, asymptotes	▪ How do we sketch a graph of a rational function? ▪ How do we represent behavior around asymptotes with limit notation?
Solving rational equations, extraneous solutions	▪ How do we solve rational equations?
Modeling with rational functions	▪ How do we know when to model with rational functions? ▪ How do we solve the corresponding equation?

Standards

- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Perform arithmetic operations with complex numbers.

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1+\sqrt{3}i\right)^3 = 8$ *because* $\left(-1+\sqrt{3}i\right)$ *has modulus 2 and argument 120°.*

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

N-CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Use polynomial identities to solve problems.

A-APR.5 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Rewrite rational expressions.

A-APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-APR.7 ?
A-REI.8 ?
A-REI.9 ?

Analyze functions using different representations.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **F-IF.7d** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★

Pacing:

# of Days	**Topic**	**Scope of Standard**	**Textbook Section(s)**
1	Modeling with Linear, Quadratic, and Power Functions	▪ Review modeling with linear and quadratic functions. ▪ Model scenarios with power functions.	2.1, 2.2
1.5	Graphing Higher Degree Polynomials	▪ Review graph sketching of higher degree polynomials in factored and standard form. ▪ Use limit notation for end behavior. ▪ Explore the Intermediate Value Theorem.	2.3
2.5	Finding real zeros of polynomial functions, remainder/factor theorem , rational zero theorem, upper/lower bound tests	▪ Review long and synthetic division, remainder/factor theorem, what a zero is numerically, graphically, and algebraically. ▪ Explore the rational zero theorem, upper/lower bound tests	2.4
1	Operations with complex numbers complex plane, graph numbers on the complex plane.	▪ Review operations with complex numbers ▪ Graph complex numbers on the complex plane. ▪	2.5
2.5	Finding complex zeros of polynomial functions, Fundamental Theorem of Algebra, Linear Factorization Theorem, Complex Conjugate zeros	▪ Extend knowledge of finding real zeros to include finding complex zeros (complex conjugates) of polynomial functions. ▪ Develop and understand the Fundamental Theorem of Algebra and Linear Factorization Theorem.	2.6
2	Graphing and analyzing graphs of rational functions, limits, asymptotes	▪ Use characteristics from Chapter 1 to analyze graphs of rational functions. ▪ Explore horizontal, vertical, and slant asymptotes in rational functions. ▪ Use limit notation for horizontal asymptotes/end behavior and vertical asymptotes.	2.7
1	Solving rational equations, extraneous solutions	▪ Solve rational equations algebraically. ▪ Determine validity of solutions	2.8
1.5	Modeling with rational functions	▪ Model scenarios with rational functions. ▪ Solve graphically and algebraically.	2.8
2	Review and Assessment		Chapter 2

Time Period 3: Conic Sections

5-7 days + 2 days for review and testing

Notes:

- Prior Knowledge: Students should finish Geometry with an ability to derive and apply the equation of a circle and parabola.
- Precalculus Focus: Students expand their knowledge of conics to include the derivation of the equation of ellipses and hyperbolas. Students will then use these equations and graphs of conic sections to model real-world problems.
- Next Steps:
  - AP Calculus: Students will need to write an equation of a circle given a graph and graph a circle given the equation.
  - SAT Subject Test Math Level II: Coordinate geometry comprises approximately 10-14% of this test. Topics in this unit that can be categorized as coordinate geometry include: translate between the algebraic and graphical representation of circles, parabolas, ellipses, and hyperbolas and determine axes, foci, vertices, and asymptotes of conic sections from algebraic and graphical representations.

Essential Concepts	Essential Questions
Overview of Conic Sections	▪ Where do conic sections come from?
Parabolas	▪ How can we translate between the graphical and algebraic representation of a parabola? ▪ How can we identify the key features of a parabola from an equation and graph?
Ellipses	▪ How can we translate between the graphical and algebraic representation of an ellipse? ▪ How can we identify the key features of a ellipse from an equation and graph?
Hyperbolas	▪ How can we translate between the graphical and algebraic representation of a hyperbola? ▪ How can we identify the key features of a hyperbola from an equation and graph?

Standards

- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Understand and apply theorems about circles.
G-C.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Translate between the geometric description and the equation for a conic section.
G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
G-GPE.MA.3a (+) Use equations and graphs of conic sections to model real-world problems. ★

G-GMD.2 ?
G-GMD.4 ?

Pacing:

# of Days	Topic	Scope of Standard	Textbook Section(s)
0.5	Overview of Conic Sections	▪ Develop mental framework for four primary conic sections.	
2	Parabolas	▪ Review geometric definition of a parabola. ▪ Determine focus, directrix, axis of symmetry, and focal width of parabolas with vertex $(h, \square)$. ▪ Translate between algebraic and graphical representation of parabolas.	
2	Ellipses	▪ Derive geometric definition of an ellipse. ▪ Determine focal axis, foci, vertices, semitransverse axis, semiconjugate axis, Pythagorean relation, and asymptotes of hyperbolas with vertex $(h, \square)$. ▪ Translate between algebraic and graphical representation of hyperbolas.	
2	Hyperbolas	▪ Derive geometric definition of a hyperbola. ▪ Determine focus, directrix, axis of symmetry, and focal width of parabolas with vertex $(h, \square)$. ▪ Translate between algebraic and graphical representation of hyperbolas.	
1.5	Review and Assessment		Chapter 8

Time Period 4: Exponential, Logistic, and Logarithmic Functions

8-10 days + 2 days for review and testing.

Notes:

- Prior Knowledge: Students should have a strong understanding of exponential and logarithmic functions from Algebra II.
- Precalculus Focus: Students will expand their understanding to include a strong working facility with exponential and logarithmic functions in base e. Students will also explore logistic growth functions numerically, algebraically, and graphically.
- Next Steps:
    - AP Calculus: Students should be able fluently write and solve exponential and logarithmic equations with the use of exponential and logarithmic properties both with and without the use of technology. Students aiming to continue onto AP Calculus BC will need to extend this fluency to include logistic functions as well.
    - SAT Subject Test Math Level II: Functions and Algebra comprises approximately 48-52% of this test. Topics in this unit that can be categorized as functions and algebra include: exponential functions and logarithmic functions.

Essential Concepts	**Essential Questions**
Writing and modeling with exponential functions and logistic growth functions	▪ How do we know when to model with exponential and logistic growth functions?
Solving exponential equation	▪ How do we solve exponential equations (with and without technology)?
Overview of Logarithms	▪ What is a logarithm? Base e?
Properties of logarithms, solving exponential and logarithmic equations (base e)	▪ How do we solve exponential and logarithmic equations with base e? ▪ How can we apply the properties of logarithms to solve exponential and logarithmic equations?
Modeling with exponential and logarithmic functions	▪ How do we know when to model with exponential and logarithmic functions?
Mathematics of Finance	▪ How is interest calculated on various types of accounts?

Standards

- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Build new functions from existing functions.

F-BF.4 Find inverse functions.

  F-BF.4b (+) Verify by composition that one function is the inverse of another.

  F-BF.4c (+) Read values of an inverse function from a graph or a table given that the function has an inverse.

F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Pacing:

# of Days	Topic	Scope of Standard	Textbook Section(s)
1.5	Writing and modeling with exponential functions and logistic growth functions	▪ Review writing and modeling with exponential functions ▪ Explore writing and modeling with logistic growth functions	3.1
1.5	Solving exponential equation	▪ Review how to solve an exponential equations graphically and algebraically. ▪ Develop fluency in solving exponential equations without technology.	3.2
1.5	Overview of Logarithms	▪ Review logarithms as inverses of exponential functions. ▪ Develop understanding and facility in working with base e. ▪ Apply graph transformations to graphs of logarithms. ▪ Write and model with logarithmic equations (base e).	3.3
2	Properties of logarithms, solving exponential and logarithmic equations (base e)	▪ Extend knowledge of logarithmic properties to include ln. ▪ Parallel properties of logarithms as they relate to properties of exponents. ▪ Solve more complex exponential and logarithmic equations.	3.4
1.5	Modeling with exponential and logarithmic functions	▪ Apply exponential and logarithmic equations solving skills to orders of magnitude and Newton's Law of Cooling.	3.5
1	Mathematics of Finance	▪ Compounding interest (if time)	3.6
2	Review and Assessment		Chapter 3

Time Period 5: Trigonometric Functions

10-12 days +2 days for review and testing.

Notes:

- Prior Knowledge: From Geometry, students should know demonstrate facility in solving right triangles using SOH CAH TOA. From Algebra II, students should understand radian measure and explain how the unit circle enables the extension of trigonometric functions to all real numbers.
- Precalculus Focus: Students expand their knowledge of the unit circle to include the concepts of periodicity and symmetry. Students use their knowledge of inverse functions and trigonometry to develop the inverse trigonometric functions.
- Next Steps:
  - AP Calculus: Students MUST know the unit circle values and the graphs of the nine (9) trigonometric functions.
  - SAT Subject Test Math Level II: Trigonometry (including trigonometric functions and analytic trigonometry) comprises approximately 12-16% of this test. Topics in this unit that can be categorized as functions and algebra include: right triangle trigonometry, the main, reciprocal, and inverse trigonometric functions, radian measure and special right triangles. Functions and Algebra comprises approximately 48-52% of this test. Topics in this unit that can be categorized as functions include: trigonometric functions, inverse trigonometric functions, and periodic functions.

Essential Concepts	**Essential Questions**
Angle measures in degrees and radians, arc length	▪ How do we convert between degrees and radians? ▪ How do we find arc length?
Modeling with main and reciprocal trigonometric functions	▪ How do we find the main and reciprocal trigonometric functions for a right triangle?
Extending Trigonometry to Circular Functions	▪ How do we find main and reciprocal trigonometric functions for all angles?
Inverse Trigonometric Functions	▪ How do we find angles given the ratio between sides?
Modeling triangles with trigonometry	▪ How can we model triangle scenarios with trigonometric functions for right and non-right triangles?
Graphs of Sine and Cosine	▪ How are the graphs of sine and cosine related? ▪ How can we apply knowledge of graph transformations to these graphs?
Graphs of Secant, Cosecant, Tangent, and Cotangent	▪ How can we develop the graphs of secant, cosecant, tangent, and cotangent from the graphs of sine and cosine? ▪ How can we apply knowledge of graph transformations to these graphs?
Graphs of Composite Trigonometric Functions	▪ What information can we use to predict the behavior of combined algebraic/trigonometric functions?

Standards
- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Build new functions from existing functions.

F-BF.4 Find inverse functions.

F-BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

Extend the domain of trigonometric functions using the unit circle.

F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

Pacing:

# of Days	Topic	Scope of Standard	Textbook Section(s)
1	Angle measures in degrees and radians, arc length	▪ Review arc length. ▪ Review angle measures in degrees and radians.	4.1
1.5	Modeling with main and reciprocal trigonometric functions	▪ Review sine, cosine, and tangent. ▪ Develop understanding of reciprocal trigonometric functions. ▪ Model triangles with six trigonometric functions.	4.2
2	Extending Trigonometry to Circular Functions	▪ Review unit circle. ▪ Deepen understanding of where values come from. ▪ Develop high level of fluency in using the unit circle from memory.	4.3
1	Inverse Trigonometric Functions	▪ Develop, analyze and apply inverse trigonometry.	4.7
1	Modeling triangles with trigonometry	▪ Review SOH CAH TOA. ▪ Review Law of Sines and Law of Cosines.	4.2, 4.8
1	Graphs of Sine and Cosine	▪ Review graphs of sine and cosine. ▪ Apply knowledge of graph transformations to sinusoids.	4.4
2	Graphs of Secant, Cosecant, Tangent, and Cotangent	▪ Use graphs of sine and cosine to develop graphs of secant, cosecant, tangent, and cotangent. ▪ Apply knowledge of graph transformations.	4.5
1	Graphs of Composite Trigonometric Functions	▪ Develop understanding of periodicity and sinusoids ▪ Use knowledge of functions to analyze functions that are combinations of algebraic and trigonometric functions.	4.6
2	Review and Assessment		Chapter 4

Time Period 6: Analytic Trigonometry

9-11 days + 2 days for review and testing.

Notes:

- Prior Knowledge: From Geometry, students should know demonstrate facility in solving non-right triangles using the Law of Sines and the Law of Cosines. They should also have exposure to the derivation of the formula $A = \frac{1}{2}ab\sin C$. From Algebra II, students should be able to prove the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.
- Precalculus Focus: Students expand their work with trigonometric proofs to include reciprocal, cofunction, odd/even, sum/difference, and multiple angle identities. Students must prove the addition and subtraction identities for sine, cosine, and tangent and use them to solve problems.
- Next Steps:
  - AP Calculus: Students pursuing AP Calculus BC should know the double-angle identities. Students aiming to take AP Calculus AB should know the reciprocal, cofunction, odd/even, and Pythagorean identities.
  - SAT Subject Test Math Level II: Trigonometry (including trigonometric functions and analytic trigonometry) comprises approximately 12-16% of this test. Topics in this unit that can be categorized as trigonometry include: trigonometric identities, Law of Sines, Law of Cosines, double angle formulas, solve trigonometric equations.

Essential Concepts	**Essential Questions**
Fundamental Identities	▪ Where do the fundamental identities come from and how do we use them to simplify trigonometric expressions and solve trigonometric equations?
Proving Trigonometric Identities	▪ What is a trigonometric identity and how do we prove one? ▪ What simplification strategies can we use to prove a trigonometric identity?
Sum and Difference Identities	▪ Where do the sum/difference identities come from and how do we use them to determine exact trigonometric values, simplify trigonometric expressions and solve trigonometric equations?
Multiple-Angle Identities	▪ Where do the double-angle, half-angle, and power-reducing identities come from and how do we use them to simplify trigonometric expressions and solve trigonometric equations?
Law of Sines	▪ How do we know when to use Law of Sines to model a triangle? ▪ How can we determine whether there are two, one, or no triangles?
Law of Cosines	▪ How do we know when to use Law of Cosines to model a triangle? ▪ How can we find the area of any triangle given SAS and/or SSS?

Standards

- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Prove and apply trigonometric identities.

F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Apply trigonometry to general triangles.

G-SRT.9 (+) Derive the formula $A = ½ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Pacing:

# of Days	Topic	Scope of Standard	Textbook Section(s)
2	Fundamental Identities	▪ Learn reciprocal, quotient, cofunction, odd-even, and Pythagorean identities. ▪ Use fundamental identities to simplify trigonometric expressions. ▪ Solve trigonometric equations.	5.1
2	Proving Trigonometric Identities	▪ Prove algebraic and trigonometric identities. ▪ Use knowledge of fundamental identities and algebraic simplification. ▪ Develop high level of algebraic and trigonometric fluency.	5.2
2	Sum and Difference Identities	▪ Prove sum/difference identities for sine, cosine, and tangent. ▪ Apply sum/difference identities to find exact values without technology. ▪ Prove trigonometric identities using sum/difference identities and fundamental identities	5.3
2	Multiple-Angle Identities	▪ Prove double-angle, half-angle, and power-reducing identities for sine, cosine, and tangent. ▪ Apply double-angle, half-angle, and power-reducing identities to find exact values without technology. ▪ Prove trigonometric identities using double-angle, half-angle, and power-reducing, sum/difference identities and fundamental identities	5.4
1	Law of Sines	▪ Review Law of Sines. ▪ Extend understanding to include ambiguous cases. ▪ Model real-world scenarios.	5.5
1	Law of Cosines	▪ Review Law of Cosines. ▪ Derive and apply the area of a triangle formula (SAS). ▪ Apply the area of a triangle formula (SSS). ▪ Model real-world scenarios.	5.6
2	Review and Assessment		Chapter 5

Time Period 7: Vectors, Parametric, and Polar Equations

10-12 days + 2 days for review and testing.

Notes:

- Prior Knowledge: In rare scenarios, students may have some exposure to vectors from Algebra II. Students may also have more exposure to vectors in Physics but the scope of that exposure is currently unknown. It should be the general rule of thumb that students will have minimal prior exposure to vectors and little-to-no prior exposure to parametric and polar functions.
- Precalculus Focus: Students will develop an understanding of vectors and their applications to real-world problems. Students will also explore how parametric equations can be used to model real-world scenarios. Finally, students will learn how to convert between rectangular and polar coordinates and describe and graph polar functions. If there is time, students should explore De Moivre's Theorem to help them find n^{th} roots of complex numbers.
- Next Steps:
    - AP Calculus: Students continuing on to AP Calculus BC will need to apply their knowledge of vectors, parametric, and polar equations for differential and integral calculus.
    - AP Science: Students will need knowledge of vectors and parametric equations for AP Physics.
    - SAT Subject Test Math Level II: Coordinate geometry comprises approximately 10-14% of this test. Topics in this unit that can be categorized as coordinate geometry include: expressing points in rectangular and polar form. Functions and Algebra comprises approximately 48-52% of this test. Topics in this unit that can be categorized as functions and algebra include: recognize the graph of a parametric function and determine its domain.

Essential Concepts	**Essential Questions**
Vectors in the plane	▪ How do we determine the magnitude and direction of a vector? ▪ How do we perform addition/subtraction and scalar multiplications of vectors? ▪ How do we find unit vectors and direction angles of two vectors? ▪ How do we use vectors to model real-world scenarios?
Dot product of vectors	▪ How do we find the dot product of two vectors? ▪ How can we use dot products to determine the angle between two vectors, the length of one vector, determine whether two vectors are orthogonal or not, and vector projection? ▪ How do we use vectors to model real-world scenarios?
Parametric equations and motion	▪ How can we use parametric equations to model motion? ▪ How do we eliminate the parameter of time?
Polar coordinates	▪ How do we convert between rectangular and polar coordinates?
Graphs of polar equations	▪ How can we determine symmetry of polar graphs? ▪ How are graphs of polar equations useful for calculus?

Standards

- Standards with a star ★ indicate Modeling standards, connected to Math Practice Standard 4.
- Standards with a plus sign (+) indicate a standard beyond College and Career Ready.

Represent complex numbers and their operations on the complex plane.

N-CN.4 (+) Represent complex numbers on a complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Represent and model with vector quantities.

N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).

N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

N-VM.4 (+) Add and subtract vectors.

N-VM.4a (+) Add vectors end-to-end, component-wise, and by the parallelogram rule.

N-VM.4b (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

N-VM.4c (+) Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5 (+) Multiply a vector by a scalar.

N-VM.5a (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

N-VM.5b (+) Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

N-VM.6 ?
N-VM.7 ?
N-VM.8 ?
N-VM.9 ?
N-VM.10 ?
N-VM.11 ?
N-VM.12 ?

Pacing:

# of Days	**Topic**	**Scope of Standard**	**Textbook Section(s)**
2-3	Vectors in the plane	▪ Define vectors and write vectors in component form. ▪ Determine the magnitude of a vector. ▪ Learn addition/subtraction and scalar multiplication for vectors. ▪ Determine a unit vector. ▪ Determine the direction of a vector by stating its direction angle. ▪ Model real-world scenarios with vectors (gravity, wind, etc.).	6.1
2	Dot product of vectors	▪ Use dot product of two vectors to calculate the angle between two vectors, the length of one vector, determine whether two vectors are orthogonal or not, and vector projection. ▪ Model real-world scenarios with vectors (gravity, wind, etc.).	6.2
2.5	Parametric equations and motion	▪ Use parametric functions to model real-world scenarios. ▪ Find parametric equations for lines. ▪ Use parametric functions to simulate motion with a grapher.	6.3
1.5	Polar coordinates	▪ Define and plot points in the polar coordinate system. ▪ Convert between rectangular and polar coordinates. ▪ Find distance using polar coordinates.	6.4
2	Graphs of polar equations	▪ Extend understanding of symmetry about the x-axis, y-axis, and origin to polar graphs. ▪ Analyze polar graphs (including rose, limacon, and lemniscates curves)	6.5
2	Review and Assessment		Chapter 6