What students need to know for...
Functions, Statistics & Trigonometry (FST) 2016-2017 NAME:_____

This is a **MANDATORY** assignment that will be **GRADED**. It is due the first day of the course. Your teacher will determine how it will be counted (i.e. homework, quiz, etc.)

Students expecting to take **Functions, Statistics & Trigonometry** at Cambridge Rindge and Latin High School should demonstrate the ability to...

General

- Keep an organized notebook
- Be a good note taker
- Complete homework every night
- Be active learners
    - ask questions and participate in class
    - seek help outside of class if needed
- Work with others
- Work with and without a calculator

Specific Math Skills

- Work with basic statistical quantities – mean, median, mode, range
- Basic operations with real numbers
- Fractions, Decimals and Percentages
- Order of operations – PEMDAS
- Evaluate algebraic expressions using substitution
- Work with algebraic expressions
    - Identify like terms
    - Add/subtract like terms
    - Multiply/divide polynomials using the distributive property
- Solve equations in one variable
    - One-step/Two-step/multi-step
- Knowledge of the Cartesian coordinate plane
    - Plotting points
    - Knowledge of points in relation to the quadrants
- Graph linear equations
    - Slope intercept form: $y = mx + b$
    - Standard form: $Ax + By = C$
    - Using x and y intercepts
- Identify functions using multiple representations table / equations / graphically
- Work with exponents and logarithms
- Work with polynomials
- add/subtract/multiply (Distributive Property and FOIL)
- Factor polynomials including quadratics

REVIEW PROBLEMS: *NOTE: Show all of your work. Your teacher may count this as a quiz grade, a homework grade, or they may give a quiz on this material at the beginning of the year. Don't forget to use the reference sheet on page 2. **You should "Google" the topic if you are unsure how to complete the examples. Khanacademy.org has some good instructional videos.** Good luck!

- The CRLS Math Department

Procedural Checklist / Reference Sheet:

Number Sense & Operations

Finding Percentage
1. Change the percent to a decimal
2. Multiply the total amount by the decimal

Changing Fractions to Decimals
1. Divide the numerator by the denominator
2. Round to the nearest hundredth if needed

Changing Fractions to Percent
1. Divide the numerator by the denominator
2. Round to the nearest hundredth
3. Drop the decimal point
4. Add a percent sign

Solving Multi-Step Operations -- PEMDAS
1. Complete all computation inside the parenthesis, brackets, or absolute value
2. Carry out all exponents
3. Do multiplication or division, from left to right
4. Do addition or subtraction, from left to right

Distribution
1. Multiply the # or variable outside the parenthesis by each term inside the parenthesis
2. Check the signs (+/-)

Multiplying Exponents vs. Dividing Exponents

1. Add exponents	Subtract exponents
2. Multiply integers	Divide integers

Multiplying by a Fraction
1. Multiply the numerator by all values
2. Divide this product by the denominator

Estimating the value of a Radical ($\sqrt{\ }$)
1. For a square root, find the closest square number.
2. Estimate the value (higher/lower)
3. Estimate this value.

Muliplying Binomials
1. Use **FOIL** -- first, outside, inside, last
2. Use box method & combine like terms

Patterns, Relations, and Algebra

Solving Equations for One Variable
1. Distribute
2. Combine Like Terms
3. Get all the variables on the left side (+/-)
4. Get all number values on the right side (+/-)
5. Divide both sides by the coefficient
6. Remember, whatever you do to one side, you must do to the other

Using Proportional Relationships
1. Determine the Part to Whole relationship
2. Write a ratio for the KNOWN part to whole
3. Determine the second ratio -- given/missing information
4. Set up a proportion with X representing missing value in the UNKNOWN ratio

Properties of Proportions
1. If $\frac{a}{b} = \frac{c}{d}$, then ad = bc
2. product of the means = product of the extremes Cross multiply to solve for missing variable

Ratios used in Proportional Relationships
1. Part / Whole
2. Percent (%) / 100
3. # of degrees / 360
4. sample / total population
5. Part:Part

Solving Systems of Equations w / Substitution
1) +/- the x term, move to the right side
2) ÷ by the coefficient of y (÷ by # with y)
3) Set the expressions equal to each other & solve for x.
4) Substitute x & solve for y.
5) Write solution as a coordinate pair (x , y).

Using the Equation of a Line/Slope(m)

$y = mx + b$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$

Be sure to carefully read each example and review information provided BEFORE you work on each section.

Part 1: Vocabulary

Match each definition or example with the appropriate vocabulary term.

coefficient	*constant*	*denominator*	*difference*	*distributive property*	
like terms	*numerator*	*product*	*quotient*	*slope*	*percentage*
slope-intercept form	*mean*	*linear- standard form*		*sum*	*range*
x-intercept	*y-intercept*	*mode*	*median*	*probability*	

1. _____ the bottom number in a fraction
2. _____ an amount obtained by addition
3. _____ $y = mx + b$
4. _____ $Ax + By = C$
5. _____ mathematical average of all the terms in a data set
6. _____ an amount obtained by multiplication
7. _____ the point where a line crosses the *y*-axis
8. _____ a value or quantity at the midpoint of a data set
9. _____ an amount obtained by division
10. _____ $4(2x – 9) = 8x – 36$
11. _____ the top number in a fraction
12. _____ an amount obtained by subtraction
13. _____ the number being multiplied by a variable (the number in front of the variable)
14. _____ a term that has no variable factor (it is just a number)
15. _____ terms with exactly the same variable
16. _____ the ratio of a line's vertical change to its horizontal change
17. _____ the most frequently occurring value in a data set
18. _____ a rate, number, or amount in each hundred
19. _____ the likelihood of a given event's occurrence
20. _____ the difference between the lowest and highest values

Part 2: PEMDAS

Evaluate the following problems in the order described here:

1. P - Parentheses
2. E - Exponents (Powers and Square Roots)
3. MD - Multiplication and Division (Left to Right)
4. AS - Addition and Subtraction (Left to Right)

1.) $14 + 18 \div 2 \times 18 - 7$

2.) $15 \times 18 + 12 \div 3 + 9$

3.) $8 \times 4 + 9 - 9 + 18$

4.) $11 \times 11 - 6 \times 17 + 4$

5.) $2 - 1 + 5 \times 4 \times 11$

6.) $16 \times 7 \times 15 + 11 + 17$

7.) $10 - 9 \times 24 \div 8 \times 6$

8.) $10 \div 5 + 10 - 9 \times 11$

9.) $3 \times 19 \times 14 + 18 \div 2$

10.) $10 \times 12 - 14 \div 2 + 15$

Part 3: Evaluating Variable Expressions

Evaluate means to find the value of, or simplify. Substitute the given values and solve.

Example: Evaluate $6x + 2y - x$, if $x = 3$ and $y = -2$

$6x + 2y - x$	
$6(3) + 2(-2) - 3$	*Substitute*
$18 - 4 - 3$	*Simplify using the Order of Operations (PEMDAS)*
$14 - 3$	
11	*Final answer*

1.) $15 - x + y$, if $x = 9$ and $y = 11$

2.) $(6c + 2b) - 11$, if $a = 8$, $b = 5$, and $c = 3$.

3.) $3(2c + d) - d$, if $c = 5$ and $d = -1$

Part 4: Properties

Properties are what allow us to manipulate mathematical expressions. As a reminder, some properties are listed here:

Additive Identity Property	*Multiplicative Identity Property*	*Commutative Property*
Additive Inverse Property	*Multiplicative Inverse Property*	*Distributive Property*

Name the property demonstrated in each example.

1.) $(-3) + 3 = 0$

2.) $8(1) = 8$

3.) $3(x + y) = 3x + 3y$

4.) $\frac{4}{3} \cdot \frac{3}{4} = 1$

5.) $2(3) = 3(2)$

Part 5: Solving Linear Equations

To solve an equation you get the variable alone by using **opposite operations**. Once you get your solution, you can check it, by plugging in to the original equation and seeing if it makes it true.

Example: Solve *6x = 4(x + 5),* then check the solution.

6x = 4(x + 5)	
6x = 4x + 20	*Simplify by distributing*
-4x -4x	*Use opposite operations-subtract 4x to get the x's on the same side of the equation*
2x = 20	
x = 10	*Use opposite operations-divide by 2 to get x by itself*

Solve and CHECK the following equations.

1.) $5w + 1 = -19 + 3$

Check:

2.) $4v - 9 = 6v + 7$

Check:

3.) $6(y + 3) = 24$

Check:

4.) $4p - 5 + 2p = 7 + 5p + 2$

Check:

Part 6: Exponents, Radicals and Logarithms

Properties of Exponents

Zero as an Exponent
For every nonzero number a, $a^0 = 1$.

Negative Exponent
For every nonzero number a and integer n, $a^{-n} = \frac{1}{a^n}$.

Multiplying Powers with the Same Base
For every nonzero number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.

Dividing Powers with the Same Base
For every nonzero number a and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.

Raising a Power to a Power
For every nonzero number a and integers m and n, $(a^m)^n = a^{mn}$.

Raising a Product to a Power
For every nonzero number a and b and integer n, $(ab)^n = a^n b^n$.

Raising a Quotient to a Power
For every nonzero number a and b and integer n, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Simplify each expression. Do not leave any negative exponents.

1.) $(w^2k^3p^{-5})^2$

2.) $(-3x^5)(2xy^4)$

4.) $\left(\frac{a^3}{m}\right)^4$

5.) $\frac{r^3t^7}{t^5}$

Simplest Radical Form

-No perfect square factors under the radical sign
-No fractions under the radical sign
-No radicals in the denominator
***Do NOT write as a decimal!!!

Example: Write $\sqrt{27}$ in simplest radical form.

$\sqrt{27}$
$\sqrt{9}\sqrt{3}$ Find a perfect square factor (9). Rewrite as a product (9·3 = 27).
$3\sqrt{3}$ Simplify the perfect square.

Write in simplest radical form.

1.) $\sqrt{24}$ 2.) $3\sqrt{50}$ 3.) $\sqrt{17}$

Properties of Square Roots

Multiplication Property of Square Roots
For every number $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Division Property of Square Roots
For every number $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

You can only add/subtract like radicals.

Example: Simplify $3\sqrt{5} - 10\sqrt{5} + 2\sqrt{6}$

$-7\sqrt{5} + 2\sqrt{6}$ *Combine like radicals. Note:* $\sqrt{5}$ and $\sqrt{6}$ are *NOT like radicals so they cannot be combined.*

Refer to the properties above and simplify each expression.

1.) $\sqrt{5}+3\sqrt{5}$

2.) $7\sqrt{2}-3\sqrt{18}$

3.) $2\sqrt{3}\cdot 3\sqrt{15}$

4.) $\dfrac{\sqrt{20}}{\sqrt{5}}$

Properties of Logarithms

$\log_b MN = \log_b M + \log_b N$	**Product Property**
$\log_b \frac{M}{N} = \log_b M - \log_b N$	**Quotient Property**
$\log_b M^x = x \log_b M$	**Power Property**

II) Exercises

Expand the following logarithms.
Use either the power rule, product rule or quotient rule.

1. $\log_2(9^5)$ = _____

2. $\log_2(21)$ = _____

13. $\log_2\left(\dfrac{1-x}{y}\right)^3$ = _____

14. $\log_3 \sqrt[5]{9x^3}$ = _____

Part 7: Simplifying Polynomials

- You can only add/subtract like terms
- Refer to properties of exponents for multiplication
- Standard Form: write terms in descending order by degree (exponent)

Simplify each of the following. Write your answers in standard form.

1.) $(4x^2 + 2x + 5) + (7x^2 - 5x + 2)$

2.) $(9a^2 - 5a - 4) - (-6a^2 + 12a + 3)$

3.) $8b(3b + 7)$

4.) $(p + 2)(-5p + 4)$

Part 8: Summary of Factoring Techniques

Factoring is a method of writing an expression as a product of factors. For example, we can write 10 as (5)(2), where 5 and 2 are called factors of 10. We can also do this with polynomial expressions. In this section we look at several ways to factor polynomial expressions.

- For all polynomials, first factor out the greatest common factor (GCF).

- For a binomial, check to see if it is any of the following:

  1. difference of squares: $x^2 - y^2 = (x + y)(x - y)$
  2. difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
  3. sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

- For a trinomial, check to see whether it is either of the following forms:

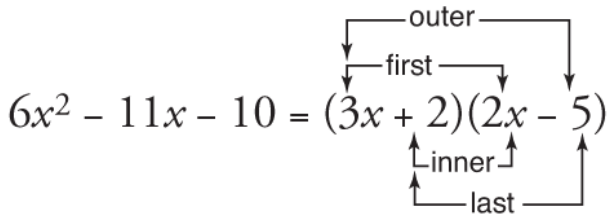
  1. $x^2 + bx + c$:
     If so, find two integers whose product is c and whose sum is b. For example,

     $x^2 + 8x + 12 = (x + 2)(x + 6)$
     since $2 + 6 = 8$ and $(2)(6) = 12$

  2. $ax^2 + bx + c$:
     If so, find two binomials so that

     - the product of first terms = ax^2
     - the product of last terms = c
     - the sum of outer and inner products = bx

See the following polynomial in which the product of the first terms = $(3x)(2x) = 6x^2$, the product of last terms = $(2)(-5) = -10$, and the sum of outer and inner products = $(3x)(-5) + 2(2x) = -11x$.



$x^2 + 2xy + y^2 = (x + y)^2$ square trinomial

$x^2 - 2xy + y^2 = (x - y)^2$

- For polynomials with four or more terms, regroup, factor each group, and then find a pattern as in steps 1 and 2.

Factor each completely.

1) $3p^2 - 2p - 5$

2) $2n^2 + 3n - 9$

3) $3n^2 - 8n + 4$

4) $5n^2 + 19n + 12$

5) $2v^2 + 11v + 5$

6) $2n^2 + 5n + 2$

Solving with the Quadratic Formula

Often, the simplest way to solve "$ax^2 + bx + c = 0$" for the value of x is to factor the quadratic, set each factor equal to zero, and then solve each factor. But sometimes the quadratic is too messy, or it doesn't factor at all, or you just don't feel like factoring. While factoring may not always be successful, the Quadratic Formula can *always* find the solution.

The Quadratic Formula uses the "a", "b", and "c" from "$ax^2 + bx + c$", where "a", "b", and "c" are just numbers; they are the "numerical coefficients". The Formula is derived from the process of completing the square, and is formally stated as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the Quadratic Formula to solve each equation. Round the solutions to the nearest hundredth.

7.) $2x^2 + 4x - 7 = 0$.

8.) $5x^2 + 13x - 1 = 0$

Part 10: Statistics and Probability

mean	The sum of a list of numbers, divided by the total number of numbers in the list.
median	"Middle value" of a list. The smallest number such that at least half the numbers in the list are no greater than it. If the list has an odd number of entries, the median is the middle entry in the list after sorting the list into increasing order. If the list has an even number of entries, the median is equal to the sum of the two middle (after sorting) numbers divided by two.
mode	For lists, the mode is the most frequent value. A list can have more than one mode.
range	The range of a set of numbers is the largest value in the set minus the smallest value in the set. Note that the range is a single number, not many numbers

1. A list of five test scores were 60, 67, 73, 63 and 67. Find the following:

   a) Mean

   b) Median

   c) Mode

2. Seven people were asked how many miles they lived from work.The responses were 15, 7, 14, 21, 5, 9 and 13. Find the following:

   a) Mean

   b) Median

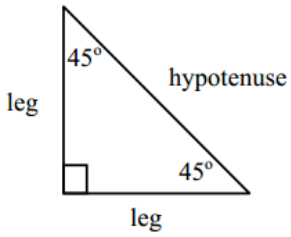
   c) Mode

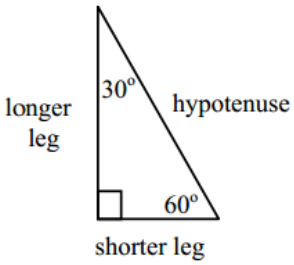
3. When a fair coin was tossed ten times, it landed heads up the first seven times. What is the probability that on the eighth toss the coin will land with tails up?
   1) $\frac{3}{10}$
   2) $\frac{1}{2}$
   3) $\frac{7}{10}$
   4) $\frac{3}{7}$

4. As captain of his football team, Jamal gets to call heads or tails for the toss of a fair coin at the beginning of each game. At the last three games, the coin has landed with heads up. What is the probability that the coin will land with heads up at the next game? Explain your answer.

Part 9: Special Right Triangles

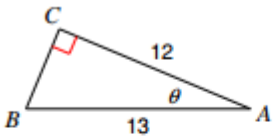
In trigonometry, we frequently deal with angle measures that are multiples of 30, 45 and 60 degrees. Because of this fact, there are two special right triangles which are useful to us as we study trigonometry. These triangles are known by the measures of their angles, and are known as 45-45-90 triangles and 30-60-90 triangles. A diagram of each is shown below:



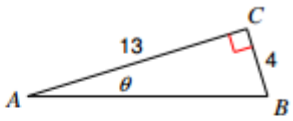


Find the measure of each angle indicated. Round to the nearest tenth.

1)



2)



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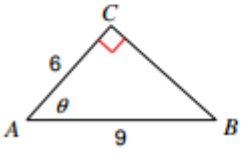

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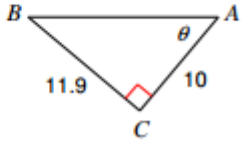

Part 10: Fractions, Decimals & Percentages

Write each as a decimal.

1) 90% 2) 30% 3) 0.3% 4) 445%

Write each as a percent.

5) 0.452 6) 0.006 7) 3.63 8) 0.03

Write each as a fraction.

9) 25% 10) 70% 11) 71% 12) 30%